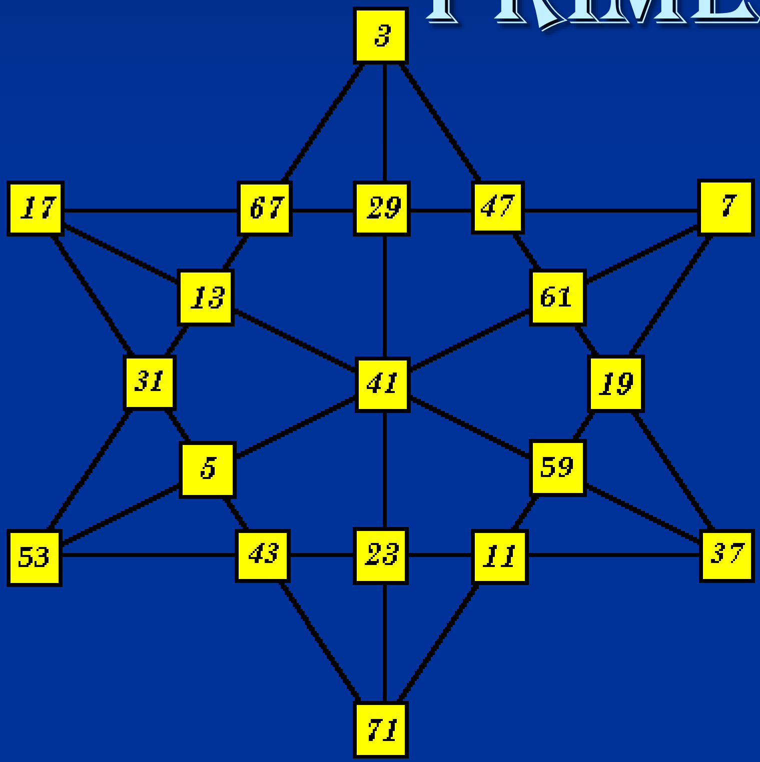
The Magic of Numbers

I. Fermat's Theorem & Prime Numbers



V. V. Raman

http://people.rit.edu/vvrsps/

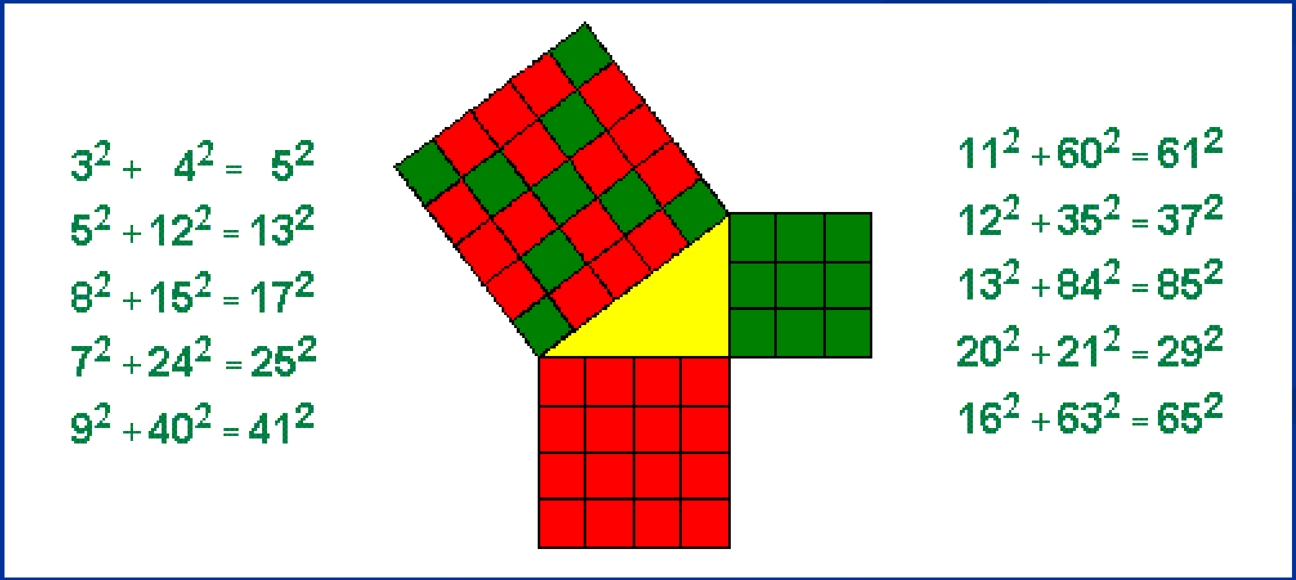
Pythagorean Triplets

Consider the three numbers 3, 4, 5.

$3^2 + 4^2 = 5^2$.

Similarly, $5^2 + 12^2 = 13^2$.

These are examples of Pythagorean triplets.



Fermat's Last Theorem

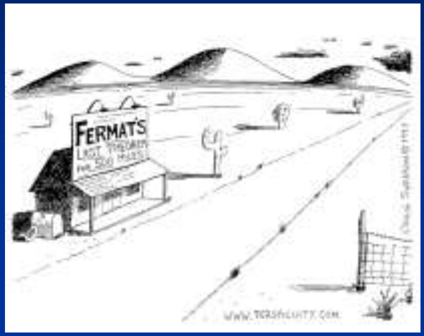
It is impossible to find integers a, b, c with

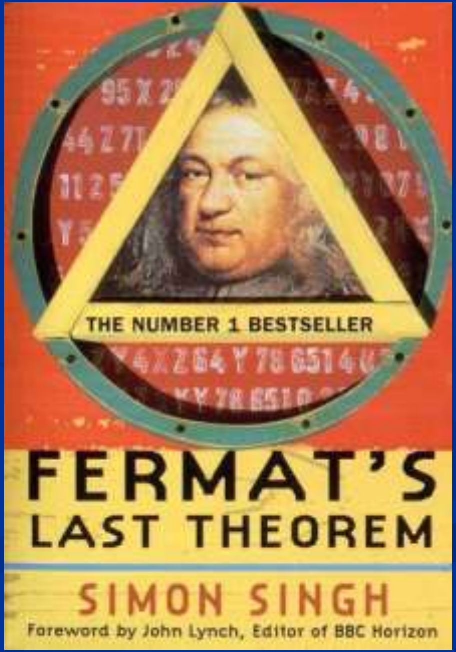
$a^n + b^n = c^n$

for n greater than 3.

i.e.there are no three numbers such that, say,

$a^5 + b^5 = c^5$.





Simon Singh's Fermat's Last Theorem

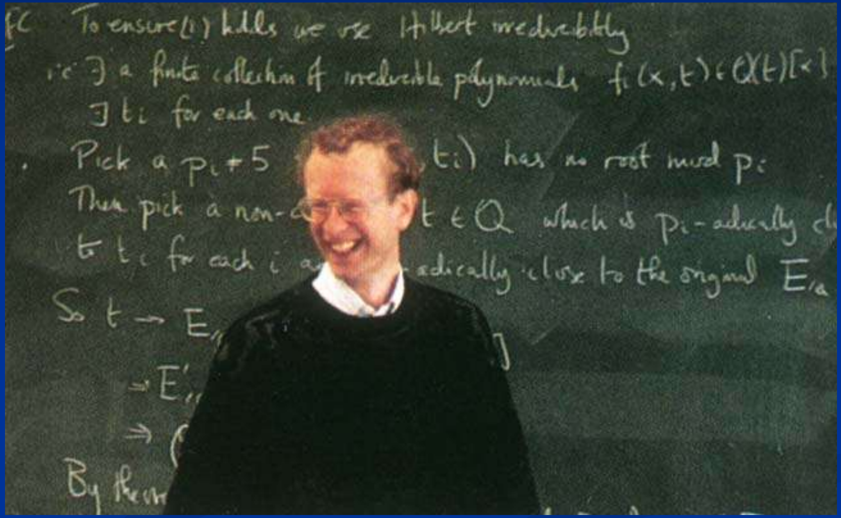
Pierre Fermat (1601 – 1665)

French Lawyer and Mathematician, contributed to many fields.

With Blaise Pascal he was one of the founders of the Theory of Probability.

Proof of
Fermat's Last Theorem

The Theorem was finally proved by Andrew Wiles in 1993: 20 years ago.



Andrew Wiles Fermat's last theorem (subtitled) - YouTube.URL

Definition of Primes

A number which has no factor other than 1 is known as a Prime Number (PN).

1 is not considered a prime.

2 is the only even prime.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

B	I	N	G	O
2	3	5	7	11
13	17	19	23	29
31	37	Free Space	41	43
47	53	59	61	67
71	73	79	83	89

How Many Primes?

There is an infinite number of PNs.

If there was a finite number of primes, then their product + 1

must also be a prime.

1x2x3x7x11x13 + 1 = 6007.

Divide this by the product 6006, you get a remainder of 1.

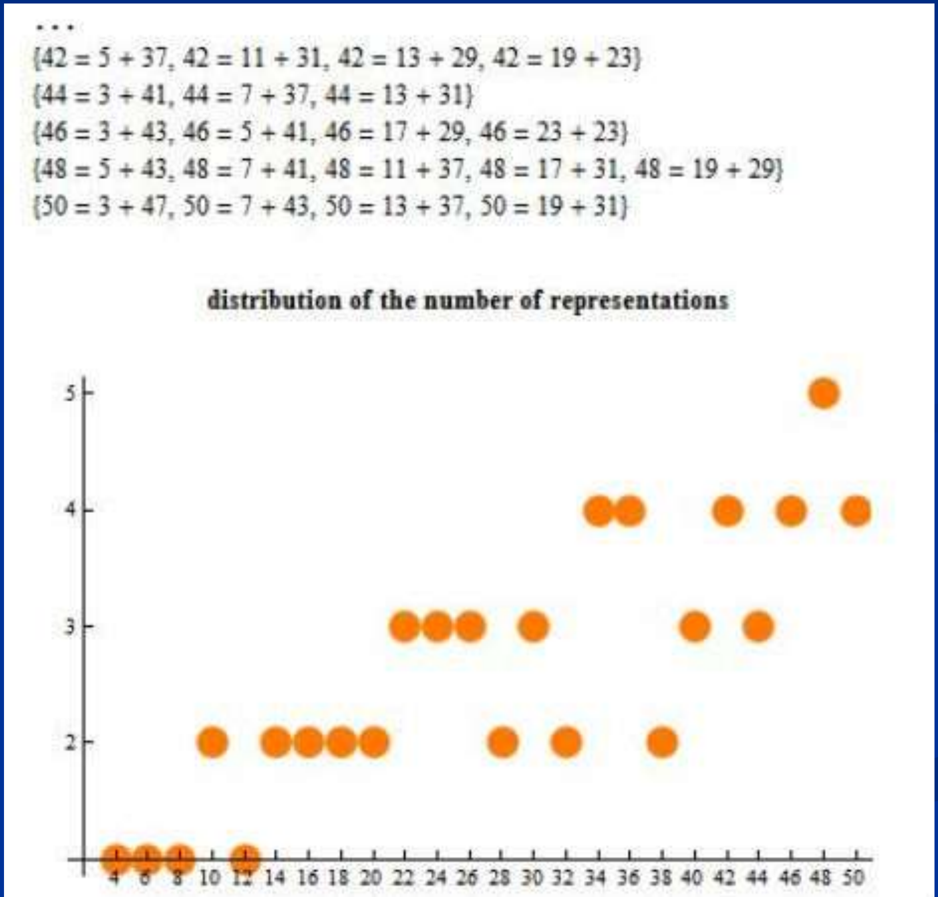
A student recently asked, "I need to know every prime number under 10^{25} for a project I have to turn in this week."

By the PNT we see this list has about 1.74×10^{23} primes on it. The student will not be completing this project. All the computer memory ever built together could not store such a list.

(Christian) Goldbach's Conjecture

Any even number E greater than 2 can be expressed as the sum of two Prime Numbers P and Q, Thus for every E,

E = P + Q.



Fermat Numbers

Take a number n. Any number in the form $2^{2^n}+1$ is called a Fermat number. If it is a prime, it is called a Fermat prime.

We are sure of only five Fermat primes.

'n'	Fermat Number = 2^(2^n) + 1	Prime?
0	3	Yes
1	5	Yes
2	17	Yes
3	257	Yes
4	65,537	Yes
5	4,294,967,297	No
6	18,446,744,073,709,600,000	No
...	...as far as has been calculated...	...No...

Mersenne Numbers and Mersenne Primes

Take any prime number p.

Then $M_p = 2^p - 1$

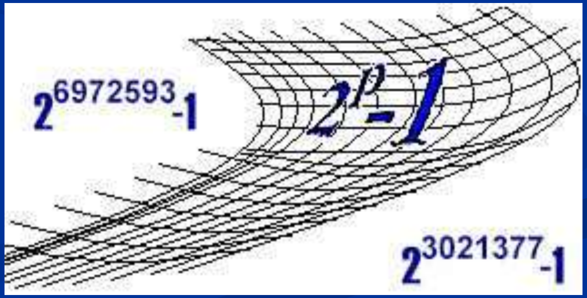
Is a Mersenne number.

A Mersenne number, not with primes, is a number of the form :

$$M_n = 2^n - 1$$

1, 3, 7, 15, 31, 63, 127, 255, (Sloane's A000225),

If it is a prime, it is a Mersenne Prime.



Thus $2^{11213} - 1$ is a Mersenne prime. Not all Mersenne numbers PN.

$M_{11} = 2^{11} - 1 = 20147 = 23 \times 89.$

Marin Mersenne (1588-1648)

He was a great thinker of the seventeenth century. He was also a priest. He was very highly regarded. He was one of the founders of the science of acoustics.

Sophie Germaine

- French mathematician who suffered gender-discrimination.
- But she worked on her own, winning a grand prize from the French Academy of Science.
- She did work on Fermat's last theorem.

Sophie's Primes

For a given p, if 2p + 1 is also a prime, Then p is said to be a Sophie Germain Prime.

Examples:

2, 3, 5, 11, 23, 29, 41, 53, 83, 89, 113, 131, 173, 179, 191, 233, ...

The largest know SGP is:

$18543637900515 \times 2^{666667} - 1$. (April 2012).

It has 200701 digits.

Twin primes

When two primes are separated by only the number 2.

(2, 3), (5, 7), (11, 13), (17, 19), … (41 & 43).

1,000,000,000,061, 1,000,000,000,063)

The largest known twin primes are:

$3756801695685 \times 2^{666669} \pm 1$

(December 2011)

Great Internet MP Search

WIMPS is an international hunt for Mersenne primes using computers. Begun in 1997, it has discovered 14 Mersenne primes.

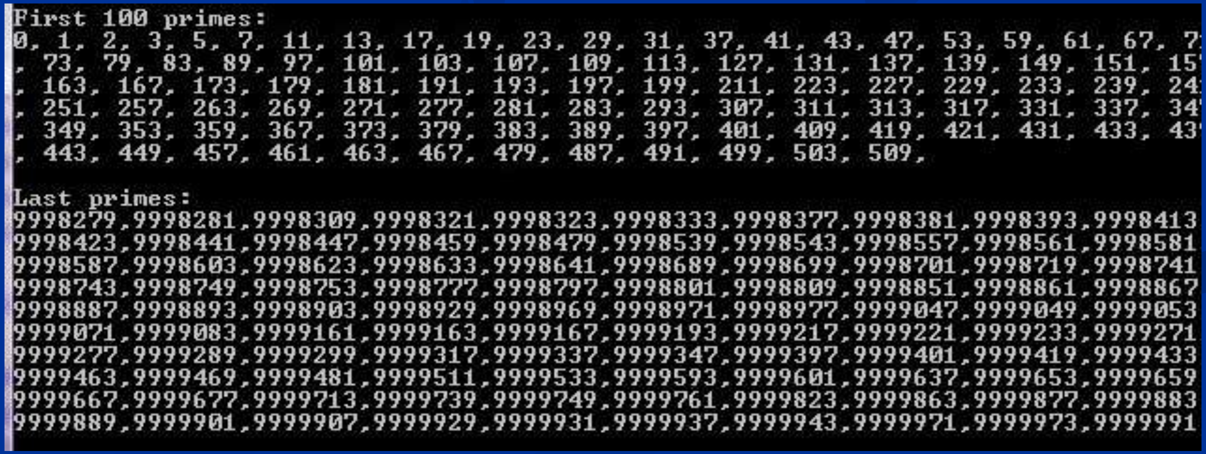
Nov 1996 M(1398269)

420921 digits

35th MPN

Feb 2013

48th MPN



Longest Prime Thus Far

Curtis Cooper, a computer science professor at the University of Central Missouri in Warrensburg.

The number is:

$$2^{57,885,161} - 1$$

longest prime number with 17,425,170 digits.

Mod (10)

Modulo (9)

Equivalent Digit (ED)

14 = 9 + 5

49 = 9.5 + 4

88 = 9.9 + 7

N = 9n + ED(N)

Mod 9: Equivalent Digit

Fractions